Linear Algebra and its Applications Workshop

A satelite workshop of

July 30th and 31st, 2018

Preface

Linear algebra plays a key role in many areas of mathematics. The Linear Algebra and its Applications Workshop (LAAW 2018) aims to discuss relevant aspects of linear algebra research and its applications. Considered topics were matrix theory, numerical linear algebra, spectral theory of graphs and combinatorics.

This volume collects the abstracts of plenary speakers, invited speakers, contributed talks and posters presented at the LAAW 2018, held at the Instituto de Matemática e Estatítica of Universidade Federal Fluminense, Niterói, Rio de Janeiro, Brazil, 30th and 31st of July, 2018.

LAAW2018 is a satellite event of ICM 2018 (International Congress of Mathematicians), that will be held in Rio de Janeiro, from August 1st to 9th, 2018.

In 2019, Rio de Janeiro will host the International Linear Algebra Symposium, ILAS2019, a traditional conference of the International Linear Algebra Society (http://ilas2019.org/). This workshop will act as a bridge between the two events: ICM2018 and ILAS2019, making it part of the biennium of Mathematics.

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Plenary Speakers

New spectral coordinates

Carlos Tomei
PUC-RJ, Brazil

Inverse problems and integrable systems are practical and theoretical contexts in which appropriate coordinates make a difference. We present such an example which has been very fruitful in both situations: for the understanding of familiar eigenvalue algorithms of the QR family and in the theory of isospectral flows, in which the symplectic structure is essentially bypassed. These coordinates take into account matrix profiles, extending substantially the better understood case of tridiagonal matrices.

Joint work with N. Saldanha (PUC-Rio), D. Torres(PUC-Rio) and R. Leite (UFES).

Discrete Quantum Walks and Graph Invariants

Chris Godsil
University of Waterloo, Canada;

In probability theory a discrete random walk is often defined by the powers of a stochastic matrix, a non-negative matrix with each row summing to 1. In quantum computing a discrete walk is specified by the powers of a unitary matrix. One difficulty is that to be useful, the unitary matrix must satisfy some structural conditions, and in practice this often means that the walk is based on a graph. I will discuss some of the ways interesting walks can be built from graphs. Of course, if a walk is based on a graph, then we might hope for interactions between properties of the graph and properties of the walk. I will discuss this too.

Comparing numbers of walks among graphs

Dragan Stevanović
Serbian Academy of Sciences and Arts, Serbia;

It is well known that the information on numbers of walks is contained in the powers of graphs adjacency matrix. An ability to compare numbers of walks of arbitrary length between two graphs then implies inequalities between spectral radii and Estrada indices of those graphs. Comparisons of numbers of walks are usually obtained through injective embeddings of the set of walks of one graph into the set of walks of another graph, although some shortcuts are allowed from time to time. In the lecture we will review comparison methods appearing in literature, and specifically those used in recent results such as the proof of Belardo-Li Marzi-Simić conjectured extension of the Li-Feng lemma, Andriantiana and Wagners result on greedy tree with given degree sequence, and ordering of starlike trees by the numbers of walks.

Iterative Linear Solvers for Oil Reservoir Simulation

José Roberto P. Rodrigues
Petrobras/Cenpes, Brazil;

Reservoir simulation is a crucial component of almost every decision in the oil production industry. At the core of these reservoir simulators is the solution of very large systems of linear equations, which is typically the step requiring the highest amount of computer resources. After briefly reviewing the techniques used to model petroleum production and how they lead to the need of solving such large linear systems, this talk will discuss the iterative methods most commonly employed, highlighting specific features which are required by some peculiar aspects of those systems. The impact of modern computer architecture on the design of the linear solvers will be emphasized. Key aspects taken into account in a dedicated solver for petroleum applications currently under development will be shown, as well as results obtained with matrices from real simulations, focusing on performance and scalability.

Invited Speakers

Recent developments on Brouwer's conjecture for the Laplacian eigenvalues of a graph

Shariefuddin Pirzada[1]

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For a simple graph $G(V,E)$ with n vertices and m edges having vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \ldots, e_m\}$, the adjacency matrix $A = (a_{ij})$ is a $(0,1)$-square matrix of order n whose (i,j)-entry is equal to 1 if v_i is adjacent to v_j and equal to 0, otherwise. If $D(G) = diag(d_1, d_2, \ldots, d_n)$ is the diagonal matrix associated to G, where $d_i = \deg(v_i)$, for all $i = 1, 2, \ldots, n$, the matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix and its spectrum is called the Laplacian spectrum (L-spectrum) of the graph G. If $0 = \mu_n \leq \mu_{n-1} \leq \cdots \leq \mu_1$ is the L-spectrum of G and $S_k(G) = \sum_{i=1}^{k} \mu_i$, $k = 1, 2, \ldots, n$ is the sum of k largest Laplacian eigenvalues of G and $d_i^*(G) = |\{v \in V(G) : d_v \geq i\}|$, for $i = 1, 2, \ldots, n$, A. Brouwer conjectured that for any k, $k = 1, 2, \ldots, n$,

$$S_k(G) = \sum_{i=1}^{k} \mu_i \leq m + \binom{k+1}{2}.$$

We discuss the progress on this conjecture, and show that Brouwer's conjecture holds for some new classes of graphs.

Keywords: Laplacian matrix of a graph, laplacian eigenvalues, Brouwer's conjecture.

References:

[1] Hilal A. Ganie, Ahmad Alghamadi, S. Pirzada, On the sum of the Laplacian eigenvalues of a graph and Brouwer's conjecture, *Linear Algebra Appl.* **501** (2016) 376-389.

[2] S. Pirzada, H. A. Ganie, On the Laplacian eigenvalues of a graph and Laplacian energy, *Linear Algebra and its Applications* **486** (2015) 454–468.

[3] S. Pirzada, H. A. Ganie, Vilmar Trevisan, Brouwer's conjecture, preprint.

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Inequalities for the largest Laplacian eigenvalue of a graph

Leonardo de Lima [1], Rodrigo Grijo [1] and Carla Oliveira [2]

[1]Federal Center of Technological Education of Rio de Janeiro ; [2]

Let G be a graph on n vertices and let $\overline{G}$ be its complement. Write $L(G) = D(G) - A(G)$ for the Laplacian matrix, where $D(G)$ is the diagonal matrix of the degrees of G and $A(G)$ is the adjacency matrix of G. Consider $\mu_1(G)$ as the largest eigenvalue of $L(G)$. In [1], Zhai *et al.* conjectured that

$$\mu_1(G) - \mu_{n-1}(G) \leq n - 1,$$

with equality if and only if G or $\overline{G}$ is isomorphic to K_1 join to a disconnected graph of order $n-1$. In the light of the Nordhaus-Gaddum type inequality, we can rewrite the conjecture as

$$\mu_1(G) + \mu_1(\overline{G}) \leq 2n - 1.$$

In this work, we address the Nordhaus-Gaddum type inequalities to the largest eigenvalue of the Laplacian matrix. In this context, we present some results of the literature and some new ones are proved. In particular, we prove that Zhai's conjecture is true for regular graphs.

Keywords: Laplacian matrix, largest eigenvalue, bounds.

References:

[1]Zhai, M. , Shu, J. and Hong, Y., On the Laplacian spread of graphs. Applied Mathematics Letters, 24(12):2097–2101, 2011.

Oral Presentation

Spectral Graph Theory

On the null structure of bipartite graphs without cycles of length multiple of 4

Daniel A. Jaume[1], Gonzalo Molina[1], and Adrián Pastine[1]

[1]Universidad Nacional de San Luis

We say that a bipartite graph without cycles of length multiple of 4 is a BC_{4k}-free graphs. In this work we study the null space of BC_{4k}-free graphs, and its relation to structural properties. We decompose them into two different types of graphs: N-graphs and S-graphs. N-graphs are graphs with a perfect matching (the order of the graph is twice its matching number). S-graphs are graphs with a unique maximum independent set. We obtain many formulas relating the independence number and the matching number of a BC_{4k}-free graph with its N-graph and its S-graph. Among other results, we show that the rank of a BC_{4k}-free graph is twice its matching number, generalizing a result for trees due to Bevis et al [1]. About maximum independent sets, we show that the intersection of all maximum independent sets of a BC_{4k}-free bipartite graph coincides with the support of its null space.

Keywords: Bipartite Graphs, Independence number, Matching number, Maximum matchings, Independent sets, Eigenvectors, Null space.

References:

[1] Bevis, Jean H and Domke, Gayla S and Miller, Valerie A. Ranks of trees and grid graphs. *J. of Combinatorial Math. and Combinatorial Computing*, 18: 109–119, 1995.

Spectrum of the Zero-Divisor Graph of a Commutative Ring

André Ebling Brondani [1], Francisca Andrea Macedo França[1] and Daniel Felisberto Traciná Filho[1]

[1]Federal Fluminense University

Let R be a commutative ring with nonzero identity. The idea of a zero-divisor graph of R was introduced by Beck in [1], where he was mainly interested in colorings of R. Our definition of a zero-divisor graph of R, denoted by $\Gamma(R)$, and the emphasis on the interplay between the graph-theoretic properties of $\Gamma(R)$ and the ring-theoretic properties of R are due to Anderson and Livingston in [2]. Given a positive integer k, be $p_1, p_2, \ldots, p_k$ integers such that $p_\ell \leq p_{\ell+1}$, $1 \leq \ell \leq k-1$, and consider the ring $R' \simeq F_{p_1} \times F_{p_2} \times \cdots \times F_{p_k}$, where F_{p_i}, $1 \leq i \leq k$, is a field with p_i elements. In this work we will show some results, obtained during the study of the structure in $\Gamma(R')$. Among them, we determine an algebraic expression for the degree of each vertex of $\Gamma(R')$ as a function of the orders of each field, and, in particular, we obtain expressions for the maximum and minimum degrees of the graph. In addition, we compute and locate the eigenvalues of $\Gamma(R')$ for some values of k.

Keywords: Graph. Commutative ring. Zero-divisor graph.

References:

[1] I. Beck. Coloring of commutative rings. Journal of Algebra, $116 : 208-226, 1988$.

[2] D. F. Anderson and P. S. Livingston. The zero-divisor graph of a commutative ring. Journal of Algebra, $217(2) : 434-447, 1999$.

On the core-nilpotent decomposition of trees

Daniel A. Jaume[1] and Rodrigo Sota[1]

[1]Universidad Nacional de San Luis

The core-nilpotent decomposition, see [2], of a symmetric matrix A of $rank(A) = r$, says that there exist pairs of real non-singular matrices Q and C such that:

$$Q^{-1}AQ = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}.$$

Where C is an $r \times r$ matrix.

In this work we show, using the null decomposition of trees, see [1], that the core-nilpotent decomposition of the adjacency matrix of any tree can be obtain directly from the tree. In other words, we give Q and C in terms of the adjacency relations of T. This implies that, for trees, the core-nilpotent decomposition can be obtain in linear time.

Keywords: Core-nilpotent decomposition, null decomposition, trees.

References:

[1] "Null decomposition of tree". Daniel A. Jaume and Gonzalo Molina. Discrete Mathematics, Volume 341, Issue 3, Pages 836-850, March 2018.

[2] "Matrix analysis and applied linear algebra", Carl D Meyer, SIAM. 2002.

Computing the determinant of the distance matrix of a bicyclic graph

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Let G be a connected graph with vertex set $V = \{1, \ldots, n\}$. The *distance* between two vertices i and j, denoted $d(i, j)$, is the number of edges of a shortest path linking them. The *distance matrix* of G is the $n \times n$ matrix such that its (i, j)-entry is equal to $d(i, j)$. Graham and Pollack [3] found a formula to compute the determinant of the distance matrix of a tree on n vertices, an acyclic and connected graph, depending only of n. Then, Bapat, Kirkland and Neumann [1] proved that the determinant of the distance matrix of a unicyclic graph, a connected graph with as many edges as vertices, depends on the length of the cycle and the number of its vertices. In an attempt to get an expression for the determinant of the distance matrix of a bicyclic graph, Gong, Zhang, and Xu [2] considered those bicyclic graphs having two edge-disjoint cycles, where a *bicyclic graph* on n vertices is a connected graph having $n + 1$ edges.

To the best of our knowledge, finding a formula for the determinant of the distance matrix of any bicyclic graph remains as an open problem. In this work, we will present new advances towards covering the remainder cases.

Keywords: bicyclic graphs, determinant, distance matrix.

References:

[1] R. Bapat, S.J. Kirkland, and M. Neumann. On distance matrices and Laplacians, Linear Algebra and its Applications 401(2005) 193–209.

[2] S-C Gong, J-L Zhang, and G-H Xu. On the determinant of the distance matrix of a bicyclic graph, arXiv:1308.2281v1.

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Spectral properties for the distance matrix of a threshold graph

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A threshold graph on n vertices can be obtained through an iterative process which starts with an isolated vertex and, at each step, an isolated vertex or a dominant vertex (a vertex adjacent to all previous vertices) is added. It can be represented by a binary sequence $(b_1, b_2, \ldots, b_n)$, where $b_i = 0$ represents the addition of an isolated vertex and $b_i = 1$ represents the addition of a dominating vertex. In this work we define a new tool from the binary sequence of a threshold graph, the binary variation ξ, as the number of indices i such that $b_i = 0$ and $b_{i+1} = 1$ in the binary sequence.

The distance matrix $D = (d_{ij})$ of a connected graph G is a real symmetric $n \times n$ matrix such that d_{ij} is the distance between vertices i and j.

In [1], Jacobs et al proved that if α is an eigenvalue of the distance matrix of a threshold graph such that $\alpha \neq -1$ and $\alpha \neq -2$, then α is simple. In this paper, we obtain the multiplicity of -2 and -1 as eigenvalues of the distance matrix, based on the binary variation of the threshold graph. Furthermore, we prove that, at least one of them, -1 or -2, is always an eigenvalue of the distance matrix of a threshold graph. As a consequence of these facts, we obtain the number of distinct eigenvalues of the distance matrix of a threshold graph.

Specially, if G is a complete split graph, different from the complete graph and the star, these results allow us to conclude that the number of distinct eigenvalues of the distance matrix of G is four.

Keywords: threshold graph, binary variation, distance eigenvalues.

References:

[1] Jacobs, David P, Trevisan, Vilmar and Tura, Fernando C, *Distance eigenvalue location in threshold graphs*,Proceedings of DGA,1–4, 2013.

A $\{-1, 0, 1\}$- and sparsest basis for the null space of a forest in optimal time

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Given a matrix, the Null Space Problem asks for a basis of its null space which is *sparsest* (i.e., has the fewest nonzeros). This problem is known to be NP-complete [3] and even hard to approximate [6]. Some heuristics for solving this problem were proposed in [2, 4, 5, 6]. The null space of a forest is the null space of its adjacency matrix. Sander and Sander [8] and Akbari et al. [1], independently, proved that the null space of each forest admits a $\{-1, 0, 1\}$-basis (i.e., a basis whose entries are -1, 0, and 1 only). Moreover, algorithms for finding one such basis for any given forest were also devised in [1, 7, 8], but the basis produced by these algorithms are not necessarily sparsest. We devise an algorithm for determining a sparsest basis of the null space of any given forest which, in addition, is a $\{-1, 0, 1\}$-basis. Our algorithm is time-optimal in the sense that it takes time at most proportional to the number of nonzeros in any sparsest basis of the null space of the input forest. Moreover, we show that, given a forest F on n vertices, the set of those vertices x for which there is a vector in the null space of F that is nonzero at x and the number of nonzeros in any sparsest basis of the null space of F can be found in $O(n)$ time.

Keywords: forest, null space basis, sparsest basis.

References:

[1] S. Akbari, A. Alipour, E. Ghorbani y G. B. Khosrovshahi. $\{-1, 0, 1\}$-basis for the null space of a forest. *Linear Algebra Appl.*, 414(2-3):506–511, 2006.
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Spectra of complementary prisms

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The complementary prism $G\overline{G}$ of a graph G is obtained from the disjoint union of the graph G and its complement $\overline{G}$ defined on a copy of the vertex set of G, by adding an edge for each pair vertices (v, v'), where v is in G and its copy v' is in $\overline{G}$. The Petersen graph $C_5\overline{C_5}$ and the corona of a complete graph $K_n\overline{K_n}$, with $n \geq 2$, are examples of complementary prisms. In this talk we prove that the Petersen graph is the unique complementary prism which is strongly regular. Furthermore, we compute the eigenpairs of adjacency, signless Laplacian and Laplacian matrix of a complementary prism $G\overline{G}$ in terms of the eigenvalues of adjacency, signless Laplacian and Laplacian matrix of G, respectively. In particular, to the complementary prisms of regular graphs are given special attention.

Lexicographic polynomials of graphs

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For a (simple) graph H and non-negative integers $c_0, c_1, \ldots, c_d$ ($c_d \neq 0$), the expression $p(H) = \sum_{k=0}^{d} c_k \cdot H^k$ is called the lexicographic polynomial in H of degree d, where the sum of two graphs is their join and $c_k \cdot H^k$ is the join of c_k copies of H^k. Here H^k is the k-th power of H with respect to the lexicographic product. The spectrum (if H is regular) and the Laplacian spectrum (in the general case) of $p(H)$ are determined in terms of the spectrum of H and c_k's. Based on this, some properties of graphs being lexicographic polynomials are deduced and their applications in constructions of infinite families of cospectral or integral graphs are shown.

Keywords: Lexicographic product, spectrum, Laplacian spectrum, cospectral graphs, integral graphs

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Integral Unicyclic Graphs

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We are interested in *integral unicyclic graphs.* A graph G is *integral* if its spectrum consists entirely of integers. A connected graph containing exactly one cycle is called *unicyclic.*

K. Balińska *et al.* noticed in their survey [1] that integral graphs are very rare and difficult to find. In particular, G. Omidi [2] studied integral graphs with few cycles. Although he gave necessary conditions for a unicyclic graph to be integral, he found no integral unicyclic graph different from the cycles C_3, C_4 and C_6.

We performed a computer search for all integral unicyclic graphs up to 21 vertices. Besides C_3, C_4 and C_6, we found only three of those graphs in this range. Interestingly, two of them are nonisormorphic cospectral. However, applying an algorithm for locating eigenvalues of unicyclic graphs [3], we obtained an infinite family of integral unicyclic graphs. We also were able to prove that several families of unicyclic graphs are non-integral.

Keywords: integral graph, unicyclic graph.

References:

[1] K. Balińska, D. Cvetković, Z. Radosavljević, S. Simić, D. Stevanović: *A survey on integral graphs.* Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat., **13** (2002), 42–65.

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An Analog of Matrix Tree Theorem for Signless Laplacians

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A spanning tree of a graph is a connected subgraph with minimum number of edges. The number of spanning trees in a graph G is given by Matrix Tree Theorem in terms of principal minors of Laplacian matrix of G. We show a similar combinatorial interpretation for principal minors of signless Laplacian Q. Also we prove that $\frac{\det(Q)}{4}$ is greater than or equal to the number of odd cycles in G, where the equality holds if and only if G is a bipartite graph or an odd-unicyclic graph.

Keywords: Signless Laplacian Matrix, Graph, Spanning Tree, Eigenvalue, Minor.

Matrix Theory

Critical ideals of graphs and applications

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Given a graph G and a set of indeterminates $X_G = \{x_u : u \in V(G)\}$, the *generalized Laplacian matrix* $L(G, X_G)$ of G is the matrix whose uv-entry is given by x_u, if $u = v$, and the negative number $-m_{uv}$ of edges between vertices u and v, otherwise. Let $\mathcal{R}[X_G]$ denote the polynomial ring over a commutative ring $\mathcal{R}$ with unity in the variables X_G, then for $1 \leq i \leq n$ the i-th *critical ideal* $I_i^{\mathcal{R}}(G, X_G)$ of G are the determinantal ideals spanned by $\langle \text{minors}_i(L(G, X_G)) \rangle \subseteq \mathcal{R}[X_G]$, where n is the number of vertices of G and $\text{minors}_i(L(G, X_G))$ is the set of the determinants of the $i \times i$ submatrices of $L(G, X_G)$.

Initially, critical ideals were defined in [4] as a generalization of the critical group, also known as sandpile group. Furthermore, the varieties associated to these ideals can be regarded as a generalization of the Laplacian and Adjacency spectra of G. Recently, in [1] there have been found relations between the zero forcing number and the minimum rank of a graph with the algebraic co-rank.

In this talk, we are going to outlook how all these concepts are related. And show few characterizations obtained in [2,3] for these parameters where cliques and stable sets play an important role.

Keywords: critical ideals, minimum rank, zero forcing number.

References:

[1] C.A. Alfaro, J. C.-H. Lin. Critical ideals, minimum rank and zero forcing number. preprint arXiv:1710.0338.

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On the skeleton of the matching polytope of a graph

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The matching polytope of a graph G, denoted by $\mathcal{M}(G)$, is the convex hull of the set of the incidence vectors of the matchings of G. The graph $\mathcal{G}(\mathcal{M}(G))$, whose vertices and edges are the vertices and edges of $\mathcal{M}(G)$, is the skeleton of the matching polytope of G. We want to answer two questions:
1) for which graphs G, the skeleton $\mathcal{G}(\mathcal{M}(G))$ is a regular graph?
2) given two non-isomorphic graphs, are their skeletons non-isomorphic?

Keywords: Matching Polytope, Regular Graphs, Graph Isomorphism.

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[5] L. Costa, A. Mohammadian, The Skeleton of the Matching Polytope of a Tree, preprint.

Average Mixing Matrix - A Quantum Walk Jewel

Gabriel Coutinho[1]
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Let A be the adjacency matrix of a graph, and consider the matrix $\widehat{M}$ which is the sum of the entry-wise squares of the orthogonal projectors onto the eigenspaces of A. This matrix enjoys many interesting properties: it is (1) rational (2) positive semidefinite (3) non-negative (4) doubly stochastic; but more importantly, (5) the average of the mixing matrices of a quantum walk defined on the graph. This talk intends to introduce this matrix and discuss some of its even deeper properties, and open questions. For example, we will show that $\widehat{M}$ is the matrix of transformation of the orthogonal projection onto the commutant algebra of A, restricted to diagonal matrices; and how to use this to find connections between the rank of $\widehat{M}$ and the automorphisms of the graph.

It will be based on the paper "A New Perspective on the Average Mixing Matrix", arxiv 1709.03591, by Chris Godsil (University of Waterloo), Krystal Guo (Universit libre de Bruxelles), Hanmeng Zhan (University of Waterloo) and myself.

Keywords: quantum walks, spectral graph theory, mixing matrix.

References:
[1] Gabriel Coutinho, Chris Godsil, Krystal Guo, Hanmeg Zhan. "A New Perspective on the Average Mixing Matrix" Arxiv 1709.03591, 2017.

Ergodicity of C_0-Markov semigroups on abstract state spaces

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In the theory of Markov chains, the transition probabilities $P(x, A)$ (defined on a measurable space $(E, \mathcal{F})$) define so-called Markov operator given by $Tf(x) = \int f(y)P(x, dy)$, which acts on L^1-spaces. The study of the entire process can be reduced to the study of the limit behavior of the corresponding Markov operator.

To study several properties of physical and probabilistic processes in abstract framework is convenient and important due to the classical and quantum cases confine to this scheme. We point out that in this abstract scheme one considers an ordered normed spaces and mappings of these spaces. Moreover, in this setting mostly, certain ergodic properties of Markov operators were considered and investigated. One of the main aims of the talk is to provide a general theory in abstract state spaces (i.e. real ordered Banach space with additive norm on the positive cone).

We will establish the equivalence of uniform and weak ergodicities of Markov C_0-semigroups of Markov operators in terms of the Dobrushin's ergodicity coefficient which is a new insight to this topic. This result allows us to investigate perturbations of uniformly mean ergodic operators. Note that perturbation bounds for uniformly asymptotical stable operators acting on commutative and matrix algebras have been studied in [4,5] and [6], respectively.

This work is the joint work with Prof. Dr. Farrukh Mukhamedov, United Arab Emirates University Department of Mathematical Sciences.

Keywords: abstract state space, Markov C_0- semigroups, Cesàro averages, Dobrushin's coefficient (ergodicity coefficient).

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Terwilliger Algebras of distance regularised graphs

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Let G be a connected graph. The *distance* between two vertices u and v, denoted $\partial(u,v)$, is the length of a shortest uv - path. For $u \in V(G)$ and $i \in \mathbb{Z}$, the collection of all vertices which are at distance i from vertex u is denoted by $\Gamma_i(u)$. A vertex $u \in V(G)$ is said to be *distance-regularised* if for each $x \in V(G)$, the following numbers $a(u,x) := |\Gamma_i(u) \cap \Gamma_1(x)|$, $b(u,x) := |\Gamma_{i+1}(u) \cap \Gamma_1(x)|$ and $c(u,x) := |\Gamma_{i-1}(u) \cap \Gamma_1(x)|$ depend only on the distance $\partial(u,x) = i$ and are independent of the choice of $x \in \Gamma_i(u)$. A *distance-regularised graph* is considered to be a connected graph in which every vertex is distance-regularised. Chris Godsil and John Shawe-Taylor introduced the concept of distance-regularised graphs. These graphs are shown to either be distance-regular or fall into a family of bipartite graphs called distance-biregular [1].

Let $\mathbb{C}$ denote the complex number field and let X denote a non-empty finite set. The $\mathbb{C}$-algebra consisting of all matrices whose rows and columns are indexed by X and whose entries are in $\mathbb{C}$ is denoted by $\mathrm{Mat}_X(\mathbb{C})$. The *standard module*, indicated by $V = \mathbb{C}^X$, is the vector space over $\mathbb{C}$ consisting of column vectors whose coordinates are indexed by X and whose entries are in $\mathbb{C}$. Consider a connected graph $G = (X, E(G))$ and fix $x \in X$. For every integer i, $0 \le i \le d$, where d is the eccentricity of vertex x, the *i-th dual idempotent of G with respect to x* is the diagonal matrix $E_i^* := E_i^*(x) \in \mathrm{Mat}_X(\mathbb{C})$ where, for every $y \in X$, the (y,y)-entry is equal to 1 if $\partial(x,y) = i$ and 0, otherwise. The *Terwilliger algebra (or subconstituent algebra) of G with respect to x* is considered to be the subalgebra $T := T(x)$ of $\mathrm{Mat}_X(\mathbb{C})$ generated by the adjacency matrix A of G and the dual idempotents of G with respect to x. A subspace W of V is said to be an *irreducible T-module* whenever $W \neq \left\{\vec{0}\right\}$, W is B-invariant for every $B \in T$ and contains no T-modules other than $\left\{\vec{0}\right\}$ and W. In this case, a scalar $r := r(W)$ is said to be the *endpoint of W* if $r = \min\left\{i : 0 \le i \le d \ \wedge \ E_i^* W \neq \left\{\vec{0}\right\}\right\}$. The T-module W is considered to be *thin* if the dimension of $E_i^* W$ is at most 1 for $0 \le i \le d$. It is well-known that there exists just one irreducible T-module with endpoint 0 which is called the trivial T-module.

The goal is to investigate the trivial T-modules of distance-regularised graphs. We will show that every distance-regularised vertex of a connected graph makes the trivial T-module thin. However, the converse of this statement is not true. In order to face up to this problem, a new characterisation of distance-regularised graphs in terms of its irreducible T-modules is given.

Keywords: Terwilliger Algebras, distance regularised graphs.

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Applications

Linear Systems, Interaction Graphs and Organised Complexity

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Systems Sciences and the theory of general systems were created in the middle of the last century with the purpose to overcome recognised limitations in our scientific intellectual framework in a way as to provide a basis for the construction of mathematically grounded theories in biology [1], extending also to what W. Weaver classified as problems of organised complexity [2, 4]. However, after an enthusiastic beginning the impetus in their development faded away in the middle of the 1980s. System sciences continued to provide insights in economy, sociology and engineering with an application drive and virtually no theoretical development. The theory of general systems flourished again in the life-sciences around year 2000 under the name of systems biology [5]. Although non-linearity is one of the more conspicuous aspects in the theory of general systems (TGS), the main reason for its only partial success in dealing with biological and organised complexity phenomena is structural and can be explained in the context of linear systems. In this talk, I shall use interaction graphs as well as examples from biology and ecology to unveil two central characteristics of (general) systems that have impeached TGS to completely fulfill its initial promises; one of which is the structural variability of systems describing biological phenomena. Interaction graphs can be readily obtained from the matrices associated with linear (dynamical) systems and they map which variable interacts, or affect, which other [3]. In this talk, I shall introduce interaction-graphs, show how they can be obtained from dynamical systems and use this knowledge to investigate why systems, as defined and studied in the theory of general systems, cannot cope with biological and organised complexity phenomena. Time permitting, some examples pointing to how properties of interaction-graphs provide information about the systems dynamical behaviour will be presented.

Keywords: organised complexity, general systems, interaction graphs, structure variation, hierarchy.

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Properties of Certain Graph Centrality Measures in Tournaments

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In this work, we study degree and eigenvector centralities, as well as a non-traditional measurement, called layer centrality. This methodology was preliminarily introduced in 2011, for non-weighted and non-directed graphs, though, herein, we study its basic attributes and extend it to directed and weighted graphs. Our purpose is to develop properties of these centralities for tournament rankings. First, we analyse which of them provide fair tournament rankings, in light of Arrow´s axioms. For that, we define coherent tournaments, those without intransitive cycles, for which there exists a fair non-dictatorial ranking system. At this point, we show that layer and degree centralities provide the same ranking for coherent tournaments, whereas the eigenvector centrality only provides the same ranking in certain cases. We also find that, for coherent tournaments, the layer centrality ranking is fair, according to Arrow´s axioms. Subsequently, we consider a numerical example, to evaluate how layer centrality, compared to degree and eigenvector, considers the centrality of neighbours. In the sports analogy, this characteristic corresponds to relativizing the importance of winnings and draws, according to the opponents´s strengths. We find that layer centrality does take the opponents´s strengths into account, although in a different way from the eigenvector centrality. Results show that layer centrality is the only one that provides fair rankings for coherent tournaments and also takes the opponent´s strength into account.

Keywords: Layer centrality, eigenvector centrality, degree centrality, tournament ranking, Arrow´s axioms, centrality of neighbours.

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Approximate inverse preconditioner with parallel setup to symmetric positive matrices

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Approximate inverse preconditioners are particularly useful in parallel architectures since their applications are done by simple matrix-vector multiplications. An example of these preconditioners is the one proposed by Benzi and collaborators, AINV [1], which generates an approximation for the $ZDZ^{-1} = A^{-1}$ factorization, where A is symmetric and positive definite. Although the application of the AINV preconditioner is parallel (matrix-vector multiplication), the proposed setup is intrinsically sequential. In this work, we propose an alternative algorithm, called

PARAINV, to compute a sparse approximation of the factorization $ZDZ^{-1} = A^{-1}$ with a massively parallel setup. We show that PARAINV presents a behavior similar to AINV when applied as a preconditioner of a conjugate gradient method [2] to solve a SPD system, which combined with the parallel setup, can outperform AINV in highly parallel environments, like the Intel® Xeon Phi™ (KNL) and GPUs.

Keywords: Preconditioners, approximate inverse, massive parallelism, KNL.

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Spectral approach to control value in space syntax

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As basic mathematical structures used to visually describe relations within a set of objects, graphs had found numerous applications in many different sciences including architecture and urbanism, where these applications form a subfield named the *space syntax*. Graphs in space syntax are used to describe adjacency relations between different kinds of spaces in a building or a settlement, while their invariants then serve to attach numerical value to various properties of spaces. One such invariant is the control value vector cv, defined as

$$cv_u = \sum_{v \in N_u} \frac{1}{d_v},$$

where d_v denotes the degree of a vertex v, while N_u denotes the neighborhood of a vertex u in a given connected graph. This equation can be understood to model a process in which each vertex (space) shares a unit resource (such as access to the space) equally among its neighbors, and the control value of a vertex then represents the total amount of resources that the vertex obtains from its neighbors. This process can be reiterated, thus dispersing the initial unit resources to farther neighbors, until an equilibrium is reached. If we denote the control value vector equivalently as

$$cv = AD^{-1}j,$$

where A and D are the adjacency and the diagonal degree matrix of the connected graph, respectively, and j is the appropriate all-one vector, then the final equilibrium distribution of resources is given by the principal eigenvector corresponding to the eigenvalue 1 of the matrix AD^{-1}, closely related to the normalized Laplacian matrix of the graph.

In the present talk we will compare and discuss the use of the control value and the principal eigenvector for various types of spaces in buildings.

Keywords: Space syntrax, control value, normalized Laplacian.

Posters

Q-Integral Graphs

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Let $G = (V, E)$ be a simple graph on n vertices. Define $N(u)$ as the set of neighbors of a vertex $u \in V$ and $|N(u)|$ its cardinality. The sequence degree of G is denoted by $d(G) = (d_1(G), d_2(G), \ldots, d_n(G))$, such that $d_i(G) = |N(v_i)|$ is the degree of the vertex $v_i \in V$ and $d_1(G) \geq d_2(G) \geq \ldots \geq d_n(G)$. Write A for the adjacency matrix of G and let D be the diagonal matrix of the row-sums of A, i.e., the degrees of G. The matrix $Q(G) = A + D$ is called the signless Laplacian or the Q-matrix of G. As usual, we shall index the eigenvalues of $Q(G)$ in non-increasing order and denote them as $q_1(G) \geq q_2(G) \geq \ldots \geq q_n(G)$, respectively. The graph G is called Q-integral if all eigenvalues of $Q(G)$ are integral numbers. For an arbitrary edge e of G, $deg(e)$ denotes an edge-degree, i.e., the number of edges adjacent to e. In 2007, Stanić [5] proved the there are exactly 172 connected Q-integral graphs up to 10 vertices and that every complete bipartite graph is Q-integral. In 2008, Simić and Stanić [4] found all Q-integral graphs with maximum edge-degree 4, and obtained only partial results for the next natural case, with maximum edge-degree 5. In 2009, Stanić [6] determined all $(2, s)$-semiregular bipartite Q-integral and gave some results concerning $(3, 4)$ and $(3, 5)$-semiregular bipartite Q-integral graphs. In 2010, Freitas *et al.* [1] defined the KK_n^j graphs and obtained conditions for them to be Q-integral. Also, in 2010, Freitas *et al.* [2] characterized all Q-integral in the classes: complete *split* graphs, multiple complete *split-like* graphs, extended complete *split-like* graphs, multiple extended *split-like* graphs and $K_{n_1} \vee (K_{n_2} \cup K_{n_3})$ where K_{n_i} $i = 1, 2, 3$ is a complete graphs on n_i vertices and Simić *et al.* [3] determined on that conditions the $(3, s)$-semiregular bipartite graphs are Q-integral. In 2013, Zhao *et al.* [8] gave a sufcient and necessary condition for complete r-partite graphs to be Q-integral, from which constructed infinitely many new classes of Q-integral graphs. In 2017, Zhang *et al.* [8] determine all the Q-integral unicyclic, bicyclic and tricyclic graphs. In this work, we determine Q-integral graphs giving certain conditions on the sequence of degrees.

Keywords: Signless Laplacian matrix, sequence degrees, $Q-$integral graphs

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Complementary spectra of graphs

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In spectral graph theory, we are interested in obtaining information about a graph G by just looking at its spectrum. To accomplish that, we associate the graph G with a matrix M and consider the eigenvalues of M, we call that the spectrum of G in relation to the matrix M. When two non isomorphic graphs have the same spectrum, we say they are cospectral, or they form a cospectral pair. We say that G is determined by its spectrum, DS for short, if only isomorphic graphs are cospectral to G.

There exist many papers that obtain cospectral graphs or determine graph families that are DS. But the general question that is not answered yet is "Wich graphs are determined by their spectrum"? A conjecture by W. Haemers states that almost all graphs are DS, even though its proof seems out of reach by present known mathematics. Thus the common approach in this regard is to study some particular questions, as "Is there some family of graphs that are DS?", "Are there arguments that make us believe one matrix determines more graphs than any other one?", etc. This talk intends to show a different spectral way of representing a graph.

Recently, Fernandes et al. [2] studied the complementary spectrum of a graph, defined as the complementary spectrum of its adjacency matrix, e.g. the λ satisfying the complementarity system

$$x \geq 0, \quad (Ax - \lambda x) \geq 0$$

$$x \perp (Ax - \lambda x)$$

where $\perp$ stands for orthogonality, for some $0 \neq x \in \mathbb{R}^n$.

The authors showed that the set of complementary eigenvalues of a graph G is composed by the different indices (spectral radiuses) of all the induced connected subgraphs of G. Seeger [1] proposed to represent a graph by its complementary eigenvalues.

We will establish families that are determined by their complementary spectrum (DCS, for short) reinforcing the idea that this spectrum is more effective when we are concerned in decreasing the number of coespectral graphs.

Seeger [1] showed that for all graphs up to 6 vertices, there are no pairs of cospectral mate. We advanced this and search whowing that for for up to 7 vertices, all graphs are DCS. We determine a few classes of graphs that are DCS.

Theorem 1: Let K_n, C_n, P_n and S_n be, respectively, the complete graph, the cycle, the path and the star on n vertices. We will call them elementary graphs. Among all graphs on n vertices, the class of elementary graphs is determined by its complementary spectrum.

Theorem 2: In relation to complementary eigenvalue, there is no pair of cospectral trees up to 14 vertex.

Theorem 3: Consider the following classes of graphs. If G and H are two graphs on n vertex in the same class, then G and H have different complementary spectrum.

(i) Complete bipartite graphs;

(ii) Double-stars;

(iii) Lollipops;

(iv) Brooms.

Keywords: Graphs, Complementary eigenvalues, cospectral.

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Structural genericity properties for Laplacian graph spectra

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Spectra of Laplacian matrices on weighted graphs have been extensively studied, to characterize topological properties of graphs, and to study dynamical processes of networks, as well as for data visualization. For collective behaviors of networks, simplicity of eigenvalues is of fondamental importance as it guarantees exponentially and uniformly fast convergence towards Synchronization in diffusively coupled networks and convergence to the stationary measure in random walks on graphs. Eigenvectors play also a very important role, for graphs partitioning and wavelets theory for graphs: notably the Fiedler graph partitioning depends on the entries of the eigenvector (called the Fiedler vector) associated to the algebraic connectivity. In this work we study generic spectral properties of weighted graphs. The novelty

of our approach is to study generic properties under the constraint of keeping the graph structure unchanged, and only slightly modifying the positive weights. We prove (see [1]) successively the two following properties:
- Simplicity of eigenvalues is a structurally generic property for the class of connected weighted undirected graphs: this means that given a graph in such a class, it is possible to perturb the existing weights of this graph to obtain simple eigenvalues.
- Having a Fiedler vector with nonzero pairwise distinct entries is a structurally generic property for the class of connected weighted undirected graphs.
The proof of these two properties relies on polynomial arguments, and on All-minors types theorems. This result can be applied to study the effects of Structural perturbations (such as adding links) on the Synchronization of networks [2].

Keywords: Spectral graph properties, Laplacian matrices, Perturbations of Eigenvalues.

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A Note On Graphs With Integer Algebraic Connectivity

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In this work, we study graphs with integer algebraic connectivity. With the purpose of establishing a relationship between the second smallest Laplacian eigenvalue and the vertex connectivity of such graphs, we verify that, for the cographs, a Laplacian integral graphs class, these invariants have the same value.

We also analyse two infinity families of Laplacian integral graphs which don't behave in this way. In the first, these parameters differ in a constant way and equal to an unit. On the other hand, in the second, this difference varies in the natural numbers set, that is, when we range graph to graph in this family, the difference between the connectivities varies in an unit.

Finally, we present two infinity families of graphs that are not Laplacian integral, in such a way that one has equal algebraic and vertex connectivities while the other has these same connectivities distinct constantly in an unit.

Keywords: Laplacian matrix, algebraic connectivity, spectral graph theory.

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A study on some classes of trees with the same algebraic connectivity

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The study of the ordering of graphs has always been an object of interest in the Spectral Graph Theory community, with emphasis on the ordering by the second smallest eigenvalue of the Laplacian matrix, called the algebraic connectivity. Yuan et al. [1] studied six classes of trees that maximize the algebraic connectivity for trees with more than 15 vertices, and in two of these classes of trees have the same algebraic connectivity.

In this work we extend the study of [1] creating two more classes of trees that maximize algebraic connectivity, and in one of them all trees have the same algebraic connectivity.

Keywords: Ordering Trees, Laplacian Matrix, Algebraic Connectivity.

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A study of A_α eigenvalues of operations on graphs

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Let G be a simple connected graph of order n and $A(G)$, $D(G)$ be the adjacency and the diagonal matrix of the degrees of G, respectively. Nikiforov [2] defined the convex linear combinations of $A(G)$ and $D(G)$ the following way

$$A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G), \quad 0 \leq \alpha \leq 1.$$

This matrix is called A_α matrix. In this work, we study the eigenvalues of A_α of graphs obtained from some operations.

Keywords: A_α-matrix, eigenvalues, operations on graphs.

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Nordhaus-Gaddum inequalities for q_2

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In this note, we consider a simple graph $G = (V(G), E(G))$ on n vertices and $e(G)$ edges, where $V(G) = \{v_1, v_2, ..., v_n\}$ is the vertex set and $E(G)$ is the edge set.

The *signless Laplacian matrix* of G is the matrix $Q(G) = D(G) + A(G)$, where $A(G)$ is the *adjacency matrix* and $D(G)$ is the *diagonal matrix* with the degree sequence on its main diagonal. It is well-known that $Q(G)$ is symmetric and positive semidefinite. The eigenvalues of $Q(G)$ are called the *signless Laplacian eigenvalues* of G, and are denoted by $q_1(G) \geq q_2(G) \geq \cdots \geq q_n(G)$.

The *complement of a graph* G is a graph G^c on the same vertex set such that two distinct vertices of G^c are adjacent if and only if they are not adjacent in G. Let $\rho = \rho(G)$ be an invariant in a graph G, we denote by $\rho^c = \rho(G^c)$ the same invariant in G^c.

This work is loosely inspired by the seminal paper of V. Nikiforov [1], where bounds for the (partial sums of) eigenvalues of graphs are obtained and numerous problems on the topic are proposed. Instead of dealing with the adjacency matrix of a graph G, here we are concerned with the singless Laplacian matrix of G. More precisely, we narrow down to investigate Nordahaus-Gaddum inequalities for its second largest eigenvalue.

In 1956, E. Nordhaus and J. Gaddum [2] gave lower and upper bounds on the sum and the product of the chromatic number of a graph and its complement, in terms of the order of the graph. Since then, any bound on the sum and/or the product of a graph invariant of G and the same invariant of G^c is called a *Nordhaus-Gaddum type inequality*. In general these inequalities are quite elegant as they reveal extremal values for a graph parameter and its complement. On the other hand, it may be quite difficult to be obtained.

A spectral graph invariant is a graph parameter defined using eigenvalues of the matrices associated with the graph, including the eigenvalues themselves. Many Nordhaus-Gaddum type inequalities involve eigenvalues of the adjacency, Laplacian and signless Laplacian matrices of graphs.

In 2013, Nordhaus-Gaddum type inequalities for graph parameters were surveyed by M. Aouchiche and P. Hansen [3], where it may be seen that relations of a similar type have been proposed for many other graph invariants, in several hundred papers.

We present here Nordhaus-Gaddum type inequalities for the second largest eigenvalue of the signless Laplacian matrix. More precisely, we prove the following results.

Theorem 1. Let G be a graph of order $n \geq 2$. Then $n - 2 \leq q_2 + q_2^c \leq 2(n-2)$.

Theorem 2. Let G be a graph of order $n \geq 4$, with $2 \leq e(G) \leq \binom{n}{2} - 2$. Then

$$n - 3 \leq q_2\, q_2^c \leq (n-2)^2.$$

We show that these bounds are best possible by presenting graphs satisfying the equality.

Keywords: Nordhaus-Gaddum inequality, Signless Laplacian matrix, Signless Laplacian eigenvalues.

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Oriented bipartite graphs with minimal trace norm

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Let G be a finite simple graph with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$. The adjacency matrix $A(G) = (a_{ij})$ of G is defined as $a_{ij} = 1$ if there is an edge between v_i and v_j and 0 otherwise. The eigenvalues of G are defined to be the eigenvalues of $A(G)$. The eigenvalues $\lambda_1, \ldots, \lambda_n$ of G, which are real numbers because $A(G)$ is a symmetric matrix, is called the spectrum of G. One interesting topic in spectral graph theory is the energy of a graph. It was introduced by I. Gutman in [3]. It is defined as $\mathscr{E}(G) = \sum_{i=1}^{n} |\lambda_i|$, where G is a graph with n vertices and $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of G ([4], [5]). One way of extending the concept of energy of graphs to digraphs is via the trace norm of its adjacency matrix [7]. Recall that if M is a $n \times n$ matrix with entries in $\mathbb{C}$, then the trace norm of M, denoted by $\|M\|_*$, is defined as $\|M\|_* = \sum_{k=1}^{n} \sigma_k$, where $\sigma_1, \ldots, \sigma_n$ are the singular values of M. In particular, if D is a digraph with n vertices and adjacency matrix A, we define the trace norm of D, denoted by $\|D\|_*$, as the trace norm of its adjacency matrix A, i.e. $\|D\|_* = \|A\|_*$. Note that when $D = G$ is a graph and $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of its adjacency matrix A, then since A is a real symmetric matrix, it easily follows that the singular values of A are $\sigma_k = |\lambda_k|$, for $k = 1, \ldots, n$. Consequently, $\|G\|_* = \sum_{k=1}^{n} \sigma_k = \sum_{k=1}^{n} |\lambda_k| = \mathscr{E}(G)$. The extremal value problem of the trace norm among all orientations of trees was settled in [1]. We address the following problem: among all orientations of a given graph, which has minimal trace norm? It will show that for bipartite graphs, the minimal trace norm is attained in sink-source orientations, i.e. orientations in which every vertex is a sink vertex or a source vertex ([6]). This is a consequence of result which relates the energy of a graph with the trace norm of its orientations. The proof strongly relies on a theorem by Ky Fan ([2]).

Keywords: graph energy, digraphs, singular values, trace norm

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Independent Sets and Support of Threshold graphs

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In this work we use the *support* to obtain properties of a threshold graph. The *support* of the graph G [3], denoted by $Supp(G)$, is defined as:

$$Supp(G) = \{v \in V(G) : \exists x \in \mathcal{N}(G) \text{ such that } x_v \neq 0\}.$$

Where $\mathcal{N}(G)$ denotes the null space of G, that is, the null space of its adjacency matrix. Understanding the support of a graph, in some sense, resembles the work of Fiedler [2], who obtained information of the graph based on the signs of the coordinates of the eigenvector. The support provides important structural properties of a graph. For example, in [3] was shown that the support of a tree is always an independent set. There are several applications of independent sets in real problems [1]. The cardinality of the maximum independent set of a graph G, denoted by $\alpha(G)$, is a very important classical parameter [5] and computing $\alpha(G)$ is an NP-hard problem [4]. In [3], they obtained a closed formula for the tree independence number that depends on the number of vertices of the support. In this work to extend the results of [3] to threshold graphs. In a more general sense, we want to see what structural properties we can obtain from the support of a threshold graph. In this attempt to extend the results of [3] to the threshold graphs we obtain some results, we will talk about the main results obtained (Proposition 1 and Theorem 2).

It is known that the support of trees is always an independent set (see [3]), also the support of a threshold graph is always an independent set as we can see in Proposition 1.

Proposition 1 *If G is a threshold graph then $Supp(G)$ is an independent set.*

The core of G, denoted by $Core(G)$, is defined: $Core(G) = \bigcup_{v \in Supp(G)} N(v)$.

The set of N-vertices of G, denoted by $V(\mathcal{G}_N)$, is given by: $V(\mathcal{G}_N) = V(G) - (Supp(G) \cup Core(G))$. In [3], they also obtain information about the trees through the core and N-vertices. Theorem 2 gives a nice way to compute the support, core and N-vertices of a threshold graph by analyzing only entries of the binary string that represents it, consequently, there is no need to compute the null space of the threshold graph to obtain the support, core, N-vertices.

Theorem 2 *Let G be a threshold graph and $0b_2b_3b_4 \cdots b_n$ its binary string. If k is the minimum index such that $b_k = b_{k-1} = 0$ then*

(i) $Core(G) = \{v_t : t > k \text{ and } b_t = 1\}$;

(ii) $V(\mathcal{G}_N) = \{v_t : t < k-1\} \cup \{v_t : t > k, b_t = 0 \text{ and } b_{t-1} = b_{t+1} = 1\}$;

(iii) $Supp(G) = V(G) \setminus (Core(G) \cup V(\mathcal{G}_N))$.

The following is an application of Theorem 2. Let G be a threshold graph and 01000101 its binary string. Note that $k = 4$, then by Theorem 2 we have $Core(G) = \{v_6, v_8\}$, $Supp(G) = \{v_3, v_4, v_5\}$ and $V(\mathcal{G}_N) = \{v_1, v_2, v_7\}$. Moreover, $Supp(G)$ is an independent set (Proposition 1).

Keywords: Null Space, Support, Threshold, Independent Sets.

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A spectral condition for pancyclic graphs

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Let $G = (V, E)$ be a graph, V and E denoting its vertex and edge set, respectively. We refer to the book [1] for notation and terminology not described in this work. A Hamilton cycle in a graph is a cycle through all the vertices of this graph. A graph is called Hamiltonian if it has at least one Hamilton cycle. The problem of deciding whether a graph is Hamiltonian is a well-known and difficult problem in graph theory. Indeed, determining whether a graph is Hamiltonian is NP-complete [4]. Recently, spectral methods have been applied to this problem. Fiedler and Nikiforov [2] gave sufficient conditions for a graph to be Hamiltonian in terms of the spectral radius of the adjacency matrix of the graph. In general, spectral tools have proved useful in the study of several difficult problems, such as the partitioning of graphs and the problem of isomorphism, for example.

In our work, we consider a concept that strengthens the concept of hamiltonicity. A graph G is said to be *pancyclic* if G contains a cycle of length ℓ for each ℓ with $3 \leq \ell \leq n$. A pancyclic graph is certainly Hamiltonian, but the converse is not true in general. We present a spectral condition that guarantees that a graph is pancyclic in terms of the largest eigenvalue of the adjacency matrix.

We may associate a graph G with different matrices and we call the *spectrum* of G (with respect to the chosen matrix) the multiset composed by the eigenvalues of the this matrix. The adjacency matrix $A(G) = (a_{ij})_{n\times n}$ of a graph with vertex set $V = \{v_1, \ldots, v_n\}$ is the matrix, where $a_{ij} = 1$ if v_i is adjacent to v_j, and 0 otherwise. Since $A(G)$ is symmetric its eigenvalues are real and may be written as $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. To derive a spectral condition, our idea was to look for a connection on classical sufficient condition and λ_1. We highlight the work of Zhao, Lin and Zhang [3], which presents a sufficient condition for a graph to be pancyclic depending on the degrees of nonadjacent vertices and on the distance between them. Due to this analysis, we present the following result. To state this result, we need to define the graph $G = K_{n-1} \odot e$ for the complete graph on $n - 1$ vertices with a pendant edge.

Theorem 3 *Let G be a graph of order $n \geq 6$ and λ_1 be the largest eigenvalue of its adjacency matrix. If $\lambda_1(G) > n-2$, then G is pancyclic graph unless $G = K_{n-1} \odot e$.*

We note that the same spectral condition implied that G is hamiltonian or $G = K_{n-1} \odot e$ in [2], so that Theorem 3 generalizes that result. Results of similar flavor were obtained for other strengthenings of hamiltonicity. For instance, Del-Vecchio, Vinagre and Pereira [5] have suffcient conditions for a graph to be hyper-hamiltonian.

Keywords: Graphs, Spectral Graph Theory, Pancyclic Graph.

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Spectrum of Matrogenic Graphs

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Unigraphs are those graphs that are the unique graphs (up to isomorphism) having their respective degree sequences. Relatively few graphs satisfy this requirement, but those that do comprise a number of interesting classes. For example, edgeless graphs and complete graphs are unigraphs. These trivial examples are included in the class of threshold graphs, which were defined by Chvátal and Hammer [1] which in turn are included in the class of matrogenic graphs which were introduced by Földes and Hammer [2]. In this work, as well as to reviewing some structural results, we will determine the spectrum of some classes of matrogenic graphs that are not threshold.

Keywords: Matrogenic graphs. Adjacency matrix. Eigenvalues.

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Spectrum and ABC Index of Benzenoid Graphs

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Let $G = (V, E)$ be a simple connected graph. The atom bond connectivity (ABC) index, defined in [1], provides a good model for the stability of linear and branched alkanes as well as the strain energy of cycloalkanes, which is defined as $ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$, where d_u denotes the degree of vertex u in G. Benzenoid graphs are the networks obtained by arranging congruent regular hexagons in the plane, so that two hexagons are either disjoint or possess a common edge. In this work, as well as to reviewing some structural results, we will determine the spectrum and the ABC index of some classes of benzenoid graphs.

Keywords: ABC index. Benzenoid graphs. Eigenvalues.

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Structure of Control in Financial Networks: an application to the Brazilian Stock Market

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The financial architecture of companies can be understood through the ownership network, in which the firms are linked together by property relations, forming complex patterns. We are interested in studying how is distributed the Brazilian financial ownership and who are the owners of control. In this paper we characterize the Brazilian property network and the financial control network for Brazil, using the property relations of listed companies in the Brazilian stock market and its shareholders. Following Turnovec [1], Chapelle and Szafarz [2], and Rotundo and D'Arcangelis [3], we discover the indirect ownership and control in these networks using matrices manipulations. Integrated ownership and control gives access to the real influence of a shareholder in a company and identify actual controllers. Centralities measures, link strength centrality and assoratativity, are computed from the matrices. We found that the indirect ownership is quite different form the direct ownership. However, some facts deserve attention: the same pattern is found, strong state presence in the network of property and prevalence of financial institutions.

Keywords: Control, Ownership, Financial System, Graph Theory.

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Approximated Inverse Preconditioners for $\mathcal{M}$-matrices with Blocks

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Preconditioners based on a sparse approximation of the inverse of a matrix are particularly convenient in parallel environments since their applications use a matrix-vector multiplication. One of these preconditioners is the BAINV, AINV (Approximate Inverse) in blocks, which constructs a block approach.

Benzi and coauthors, have proposed the BAINV, AINV with blocks, for a linear systems $Ax = b$, where A is a sparse $\mathcal{M}$-matrix with a block structure. We rigorously prove that the BAINV algorithm constructs a factorization $Z^T DZ = A^{-1}$ of the exact inverse of A.

The matrix-vector multiplication is a linear algebra kernel which is parallelizable in distributed and shared memories architectures, manycores or GPUs, and even hybrid architectures.

In the present work, we demonstrated that BAINV with the discard of some of its entries is robust and generates stable preconditioners when applied to the class of matrices $\mathcal{M}$.

Keywords: $\mathcal{M}$-matrices, AINV, Blocks.

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κ-Extensions of Graphs

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Given a connected, simple and finite graph X of diameter Δ we can construct an ordered list of level graphs $\langle X^{(0)}, X^{(1)}, \ldots, X^{(\Delta)} \rangle$ based on the distance levels of X such that each graph $X^{(\kappa)}$ has the same vertex set of X and xy is an edge in $X^{(\kappa)}$ iff x and y are at distance κ in X.

For each graph $X^{(\kappa)}$ in that list we define the κ-adjacency A_κ, κ-incidence (oriented or not) and κ-Laplacian (signless or not) matrices of X as the adjacency, incidence (oriented or not) and Laplacian (signless or not) matrices of $X^{(\kappa)}$, respectively.

In this work we define the κ-Intension and κ-Extension of X as the graphs represented by the adjacency matrices

$$A_1 + A_\kappa \text{ and } \begin{bmatrix} A_1 & A_\kappa \\ A_\kappa & A_1 \end{bmatrix},$$

respectively, where κ is different from both 0 and 1 for κ-Intensions.

Regarding the 1-Extension we present the relation between the classes of Twin Graphs and Trees: 1-Extensions of Trees are Twin Graphs. We also define the κ-Line Graphs which generalize for all distance levels the intuition behind the concept of Line Graphs.

This is a preliminary study motivated by the necessity of understanding relations between pairs of vertices in graphs regarding their distance levels, aiming applications in telecommunication networks.

Keywords: Graph Theory, κ-Extension, Twin Graphs.

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